$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$z^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$$

$$z^4 = 1+i$$

$$(z^4)^{\frac{1}{4}} = \left(\sqrt{2}e^{i\left(\frac{\pi}{4}+2k\pi\right)}\right)^{\frac{1}{4}}, \quad k = \pm 1, 0, 2$$

$$z = 2^{\frac{1}{8}} e^{i\left(\frac{\pi}{16}+\frac{k\pi}{2}\right)}$$

Show that the roots of the equation $z^5-(z-1)^5=0, z\neq i$, are $\frac{1}{2}\left(1-i\cot\frac{k\pi}{5}\right)$ where $k=1,2,3,4$.

$\cancel{z^5}+\triangle z^4+\ldots$

$$\boxed{\cancel{z^5}-(z-1)^5=0}$$

$$z^5=(z-1)^5$$

$$\left(\frac{z}{z-1}\right)^5=1 \qquad w^5=1$$

$$\left(\left(\frac{z}{z-1}\right)^5\right)^{\frac{1}{5}}=\left(e^{i(0+2k\pi)}\right)^{\frac{1}{5}},\ k=\pm2,\pm1,0$$

$$\frac{z}{z-1}=e^{i\left(\frac{2k\pi}{5}\right)}$$

$$z=e^{i\left(\frac{2k\pi}{5}\right)}(z-1)$$

$$e^{i\left(\frac{2k\pi}{5}\right)}=z\left(e^{i\left(\frac{2k\pi}{5}\right)}-1\right)$$

$$z=\frac{e^{i\left(\frac{2k\pi}{5}\right)}}{e^{i\left(\frac{2k\pi}{5}\right)}-1}$$

$z=a+ib$

$\rightarrow z^*=a-ib$

$$=\frac{e^{i\left(\frac{\cancel{2}k\pi}{5}\right)}}{\cancel{e^{i\left(\frac{k\pi}{5}\right)}}\left[e^{i\left(\frac{k\pi}{5}\right)}-e^{-i\left(\frac{k\pi}{5}\right)}\right]}$$

$$=\frac{e^{i\frac{k\pi}{5}}}{2i\sin\left(\frac{k\pi}{5}\right)}$$

$$=\frac{1}{2}\left(\frac{\cos\frac{k\pi}{5}+i\sin\frac{k\pi}{5}}{i\sin\left(\frac{k\pi}{5}\right)}\right),\ k\neq 0,5,10,\ldots$$

$$=\frac{1}{2}\left(\frac{1}{i}\cot\frac{k\pi}{5}+1\right)$$

$$=\frac{1}{2}\left(1-i\cot\frac{k\pi}{5}\right)$$

1. Show that the roots of the equation $z^6-(z+1)^6=0, z\neq -1$, are $\frac{1}{2}\left(i\cot\frac{k\pi}{6}-1\right)$ where $k=1,2,3,4,5$.

$$z^6=(z+1)^6$$
$$\left(\frac{z}{z+1}\right)^6=1$$
$$\left(\frac{z}{z+1}\right)^6=e^{i(2k\pi)},\ k=1,2,\dots,6$$
$$\frac{z}{z+1}=e^{i\left(\frac{k\pi}{3}\right)}$$
$$z=e^{i\left(\frac{k\pi}{3}\right)}(z+1)$$

$$z\left(1-e^{i\left(\frac{k\pi}{3}\right)}\right)=e^{i\left(\frac{k\pi}{3}\right)}$$

$z-z^*=2ib$

$$z=\frac{e^{i\left(\frac{k\pi}{3}\right)}}{1-e^{i\left(\frac{k\pi}{3}\right)}}$$
$$=\frac{e^{i\left(\cancel{\frac{k\pi}{3}}\right)}}{\cancel{e^{i\left(\frac{k\pi}{6}\right)}}\left(e^{-i\left(\frac{k\pi}{6}\right)}-e^{i\left(\frac{k\pi}{6}\right)}\right)}$$
$$=\frac{e^{i\left(\frac{k\pi}{6}\right)}}{2i\sin\left(-\frac{k\pi}{6}\right)}$$
$$=\left(\frac{1}{2}\right)\left(\frac{\cos\frac{k\pi}{6}+i\sin\frac{k\pi}{6}}{-i\sin\frac{k\pi}{6}}\right)$$
$$=\frac{1}{2}\left(-\frac{1}{i}\cot\frac{k\pi}{6}-1\right)$$
$$=\underline{\frac{1}{2}\left(i\cot\frac{k\pi}{6}-1\right)}$$

2. Show that the roots of the equation $\left(\frac{2z}{1-z}\right)^4 = 16, z \neq 1$, are $\frac{1}{2}\left(1 + i\tan\frac{k\pi}{4}\right)$ where $k = 1, 2, 3, 4$.

$$\left(\frac{2z}{1-z}\right)^4 = 16e^{i(2k\pi)}, \quad k=1,2,3,4$$

$$\frac{2z}{1-z} = 2e^{i\left(\frac{k\pi}{2}\right)}$$

$$\cancel{2}z = \cancel{2}e^{i\left(\frac{k\pi}{2}\right)}(1-z)$$

$$z + ze^{i\left(\frac{k\pi}{2}\right)} = e^{i\left(\frac{k\pi}{2}\right)}$$

$$z = \frac{e^{i\left(\frac{k\pi}{2}\right)}}{1+e^{i\left(\frac{k\pi}{2}\right)}}$$

$$z = \frac{e^{i\left(\cancel{\frac{k\pi}{2}}\right)}}{\cancel{e^{i\left(\frac{k\pi}{4}\right)}}\left(e^{-i\frac{k\pi}{4}} + e^{i\frac{k\pi}{4}}\right)}$$

$$= \frac{e^{i\frac{k\pi}{4}}}{2\cos\frac{k\pi}{4}}$$

$$= \frac{1}{2}\left[\frac{\cos\frac{k\pi}{4} + i\sin\frac{k\pi}{4}}{\cos\frac{k\pi}{4}}\right]$$

$$=$$

3. Show that the five complex numbers that satisfy the equation $\left(\frac{2w-i}{w}\right)^6 = 64$ are $\frac{1}{4}(i - \cot\frac{n\pi}{6})$ where $n = 1, 2, 3, 4, 5$.

$$\left(\frac{2w-i}{w}\right)^6 = 64$$

$$\left(2-\frac{i}{w}\right)^6 = 64e^{2k\pi}, \quad k=1,2,\ldots,6$$

$$2-\frac{i}{w} = 2e^{\frac{k\pi}{3}}$$

$$\frac{i}{w} = 2-2e^{\frac{k\pi}{3}}$$

$$\frac{w}{i} = \frac{1}{2-2e^{\frac{k\pi}{3}}}$$

$$\frac{w}{i} = \frac{1}{2-2e^{\frac{k\pi}{3}}}$$

$$\frac{w}{i} = \frac{1}{2\left(e^{\frac{k\pi}{6}}\right)\left(e^{-\frac{k\pi}{6}}-e^{\frac{k\pi}{6}}\right)}$$

$$= \frac{1}{2}\,\frac{1}{e^{\frac{k\pi}{6}}\left(2i\sin\left(-\frac{k\pi}{6}\right)\right)}$$

$$\frac{w}{\cancel{i}} = \frac{1}{4}\left[\frac{e^{-\frac{k\pi}{6}}}{\cancel{i}\sin\left(-\frac{k\pi}{6}\right)}\right]$$

$$w = \frac{1}{4}\left[\frac{\cos\left(-\frac{k\pi}{6}\right)+i\sin\left(-\frac{k\pi}{6}\right)}{\sin\left(-\frac{k\pi}{6}\right)}\right]$$

$$= \frac{1}{4}\left[\frac{\cos\frac{k\pi}{6}}{-\sin\left(\frac{k\pi}{6}\right)}+i\right]$$

$$=$$

4. Find the roots of the equation $z^5 = 1$, giving your answers exactly in the form $re^{i\theta}$.
Hence show that the roots of the equation $w^5 - (w - i)^5 = 0, w \neq i$ are $\frac{1}{2}\left(\cot\frac{k\pi}{5} + i\right)$, where $k = 1, 2, 3, 4$.

$z^5 = e^{i2k\pi}$, $k = \pm 2, \pm 1, 0$

$z = e^{i\frac{2k\pi}{5}}$

$w^5 = (w - i)^5$

$\left(\frac{w}{w-i}\right)^5 = 1$

$\therefore \frac{w}{w-i} = e^{i\left(\frac{2k\pi}{5}\right)}$

$w = e^{i\left(\frac{2k\pi}{5}\right)}(w - i)$

$w\left(1 - e^{i\left(\frac{2k\pi}{5}\right)}\right) = -ie^{i\left(\frac{2k\pi}{5}\right)}$

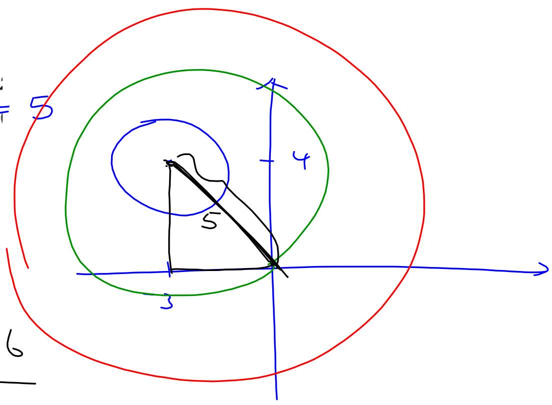
$w = \frac{-ie^{i\left(\frac{2k\pi}{5}\right)}}{1 - e^{i\left(\frac{2k\pi}{5}\right)}}$

$= \frac{-ie^{i\left(\cancel{\frac{2k\pi}{5}}\right)}}{\cancel{e^{i\left(\frac{k\pi}{5}\right)}}\left[e^{-i\left(\frac{k\pi}{5}\right)} - e^{i\left(\frac{k\pi}{5}\right)}\right]}$

$= \frac{\cancel{-i}e^{i\left(\frac{k\pi}{5}\right)}}{2\cancel{i}\sin\left(\cancel{-}\frac{k\pi}{5}\right)}$

$= \frac{\cos\frac{k\pi}{5} + i\sin\frac{k\pi}{5}}{2\sin\frac{k\pi}{5}}$

$|z-(-3+4i)| = 5$



$|z-(-3+4i)| = 6$

$|iz - 1| = \sqrt{2}$

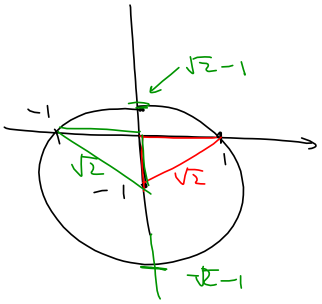
$|z - z_1| = r$

$\left| i\left(z - \frac{1}{i}\right) \right| = \sqrt{2}$

$|wz| = |w||z|$

$|i||z - (-i)| = \sqrt{2}$

$|z - (-i)| = \sqrt{2}$



$|3z + 4iz - 25| = 25$

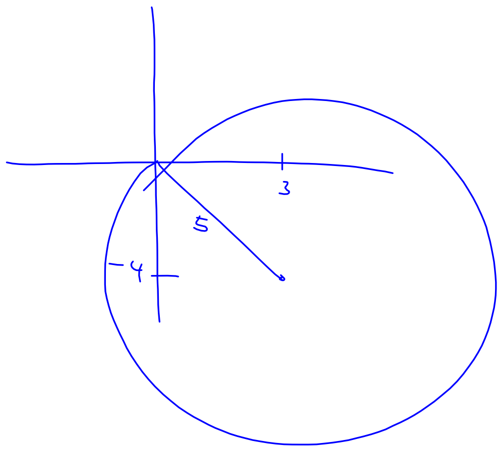
$|z - z_1| = r$

$\left| z(3+4i) - 25 \right| = 25$

$\left| (3+4i)\left(z - \frac{25}{3+4i} \right) \right| = 25$

$|3+4i| \left| z - (3-4i) \right| = 25$

$\cancel{5} \left| z - (3-4i) \right| = \cancel{25}\ 5$



1. Sketch the following loci in separate Argand diagrams:

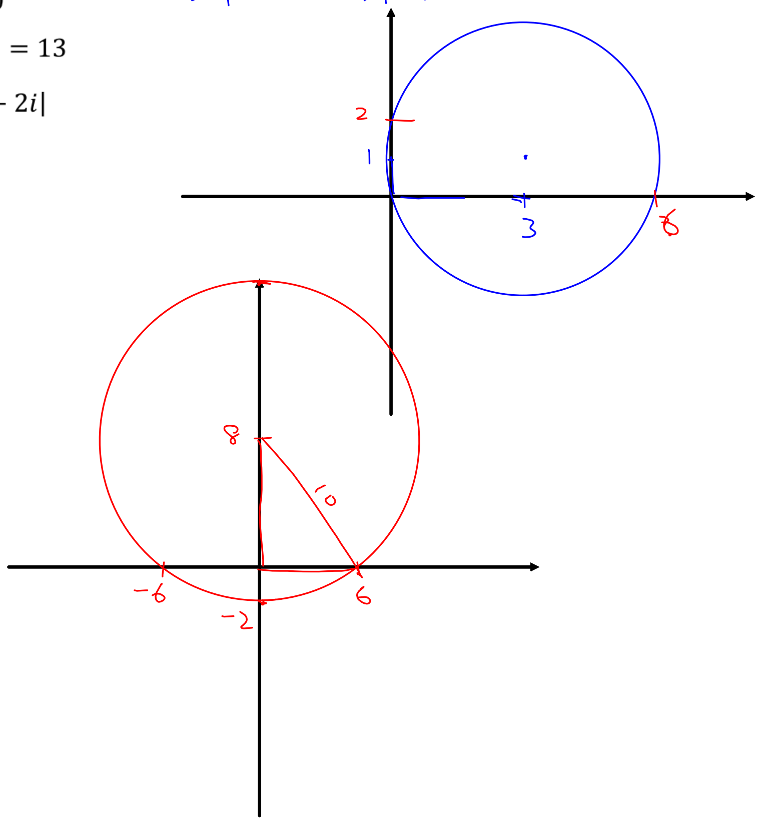
   a. $|z-3-i|=\sqrt{10}$

   b. $|z-8i|=10$

   c. $|z-5+12i|=13$

   d. $\left|\frac{z}{1+i}\right|=|-2-2i|$

a) $|z-(3+i)|=\sqrt{10}$



$\boxed{3\ \ 4\ \ 5}$

×2 6 8 10

×3 9 12 15

$\boxed{5\ \ 12\ \ 13}$

×2